Dark solitons in electron-positron plasmas

D. Farina[1] and S. V. Bulanov[2]
[1]*Istituto di Fisica del Plasma, Consiglio Nazionale delle Ricerche, Milano, Italy*
[2]*General Physics Institute of RAS, Moscow, Russia*
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We demonstrate that a dark soliton is a stable nonlinear coherent mode in electron-positron plasmas. The dark soliton comprises the minimum of the electromagnetic energy density and the minimum of the plasma density. Contrary to a bright soliton, the dark soliton can advect the trapped charged particles. The energy of trapped particles is well above the kinetic energy of the particles in the background plasma.

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The properties of coherent nonlinear structures in electron-positron plasmas have attracted a great attention in regard with the problems of astrophysical and laboratory plasmas [1]. The electron-positron plasma has been considered to be formed in the pulsar polar regions [2], in early universe [3], and the cosmological γ-ray bursts (GRB). For example, the GRB (see review [4]) are explained as to be produced in the fireballs that consist of electron-positron pairs and radiation. Multiterawatt and petawatt power laser pulses interacting with matter also produce electron-positron plasmas [5]. As in the case of the electron-ion plasma, the electron-positron plasma with the electromagnetic radiation can be a subject of various instabilities, which break up homogeneous modes into solitary waves. Often, solitons together with vortices are regarded as elementary entities comprising the turbulence. In the case of electron-ion plasmas, relativistic electromagnetic solitons have been studied analytically [6–9], and with a help of particle-in-cell simulations [10]. For the case of the electron-positron(ion) plasma the occurrence of bright soliton solutions has been discussed in details in Ref. [11]. In the electron-positron pair plasma, due to the equal electron and positron masses, novel features come into play when we describe elementary nonlinear structures. The electron-positron plasma in the pulsar magnetosphere is magnetized, but the plasma at early universe is likely to be unmagnetized (see, e.g., a discussion on the parameters in Ref. [1]). The plasma inside the GRB fireball at the initial phase of its expansion is also unmagnetized. We note that the electron-ion plasma in the field of extremely high intensity laser pulse can exhibit some properties of the electron-positron plasma.

The aim of the present paper is to investigate the properties of the relativistic solitary waves in the electron-positron pair unmagnetized cold plasmas. Exact analytical nonlinear solutions are found that show that natural coherent structures are dark solitons. Dark solitons are known in optical systems (see reviews [12], where the properties, excitation, and stability of dark solitons are discussed), and they have been observed and discussed also in the Bose-Einstein condensate [13]. In a recent paper [14], an analysis similar to the one presented here is performed taking into account thermal effects, and the solutions are obtained in a limiting case.

We use the hydrodynamics approximation to describe both the electron and positron components, and assume the plasma to be cold with zero positron and electron temperature. We consider the Maxwell's equations for the wave vector and scalar potentials $\boldsymbol{A}$ and ϕ, and the hydrodynamic equations for the density and the kinetic momentum $\boldsymbol{p}_\pm$ of positrons and electrons in the Coulomb gauge ($\boldsymbol{\nabla}\cdot\boldsymbol{A}=0$), for the one-dimensional case in which $\partial_y=\partial_z=0$. In this case, the relations $A_x=0$, and $\boldsymbol{p}_{\perp\pm}=\mp\boldsymbol{A}_\perp$ hold, where $\perp$ refers to the direction perpendicular to the x direction. We then look for a solution of the system of the form $A_\perp\equiv A_y+iA_z=a(\xi)\exp[-i\omega t+ikx+i\theta(\xi)]$ (circularly polarized wave), where a is a real quantity and the variable ξ is defined as $\xi=(x-Vt)/(1-V^2)^{1/2}$, V being velocity. The scalar potential ϕ, and the fluid quantities, i.e., the density $n_\pm$, and the parallel kinetic momentum $p_{x\pm}$ are assumed to depend only on the variable ξ. Dimensionless quantities are used. Length, time, velocity, momentum, vector, and scalar potential, and density are normalized over c/ω_{pe}, ω_{pe}, c, m_ec, m_ec^2/e, and n_0, respectively, where $\omega_{pe}=(4\pi n_0e^2/m_e)^{1/2}$ the electron plasma frequency, and n_0 the unperturbed electron (and positron) density. The following closed system of equations for the potentials is obtained, which describes coupled Langmuir and circularly polarized transverse electromagnetic (e.m.) waves,

$$\frac{d^2\phi}{d\xi^2}=V\left(\frac{\psi_-}{R_-}-\frac{\psi_+}{R_+}\right), \tag{1}$$

$$\frac{d^2a}{d\xi^2}+a\left(\overline{\omega}^2-\overline{k}^2\frac{a_0^4}{a^4}\right)=aV\left(\frac{1}{R_-}+\frac{1}{R_+}\right), \tag{2}$$

$$\frac{d\theta}{d\xi}=-\overline{k}\left(1-\frac{a_0^2}{a^2}\right), \tag{3}$$

where $\overline{\omega}=(\omega-kV)/(1-V^2)^{1/2}$, and $\overline{k}=(k-\omega V)/(1-V^2)^{1/2}$ are the Doppler shifted frequency and wave vector, respectively, and the functions $\psi_\pm=(1+a_0^2)^{1/2}\mp\phi$ and $R_\pm=[\psi_\pm^2-(1+a^2)(1-V^2)]^{1/2}$ have been introduced. The x component of the kinetic momentum, the gamma factor $\gamma_\pm=(1+p_{x\pm}^2+a^2)^{1/2}$, and the density can be expressed in terms of the potentials as $p_{x\pm}=(V\psi_\pm-R_\pm)/(1-V^2)$, $\gamma_\pm=(\psi_\pm-VR_\pm)/(1-V^2)$, $n_\pm=V(\psi_\pm/R_\pm-V)/(1-V^2)$.

To derive the above system, it has been assumed that at, e.g., $\xi=-\infty$, the electron density is equal to the positron density, the electron and positron x momentum component is zero, the electrostatic potential $\phi=0$, the electromagnetic wave amplitude a is $\pm a_0$, and the phase θ is constant. This system of equations differs from that for bright solitons mainly because there is an additional equation for the phase. Actually, in case of bright solitons the solution of Eq. (3) is simply $\theta=\theta_0-\bar{k}\xi$, so that the phase term in $A_\perp$ reads $\exp[-i\bar{\omega}\tau+i\theta_0]$, where $\tau=(t-Vx)/(1-V^2)^{1/2}$ is the proper time in the moving frame (see also Ref. [8], where a similar system was derived for the investigations of bright solitons in an electron-ion plasma with fixed ions). In the case of dark solitons, the phase behavior is in general nontrivial. Note that the system admits a first integral $(da/d\xi)^2+\bar{\omega}^2a^2+\bar{k}^2a_0^4/a^2-(d\phi/d\xi)^2/(1-V^2)+2V(R_-+R_+)/(1-V^2)=\text{const}$. Here, we are looking for localized solutions for the potentials ϕ, and a, i.e., $d\phi/d\xi=da/d\xi=0$ at infinity. It is found that $\phi=0$ is the only solution of the Poisson equation (1) with the above boundary conditions. Thus, the condition of electrical neutrality holds exactly for localized nonlinear waves in the electron-positron plasma, i.e., the relations $n_-=n_+$ and $v_{x+}=v_{x-}$ are satisfied. Using the solution $\phi=0$, from Eq. (2) we obtain a closed equation for the amplitude of the vector potential

$$\frac{d^2a}{d\xi^2}+\Omega^2a+\bar{k}^2a\left(1-\frac{a_0^4}{a^4}\right)-aF(a^2)=0, \tag{4}$$

where $F(a^2)=2V/[V^2+a_0^2-a^2(1-V^2)]^{1/2}$ and $\Omega^2\equiv\bar{\omega}^2-\bar{k}^2=\omega^2-k^2$. By imposing the constraint that $a=\pm a_0$ is a stationary solution of Eq. (4), the nonlinear dispersion relation reads

$$\Omega^2=\frac{2}{\sqrt{1+a_0^2}}. \tag{5}$$

Equation (4) has an exact solution for dark solitons of arbitrarily large amplitude. Introducing the normalized quantity $\bar{a}=a[(1-V^2)/(V^2+a_0^2)]^{1/2}$, we rewrite Eqs. (3) and (4), with Ω given by the dispersion relation, as

$$\theta'=-\beta\left(1-\frac{\bar{a}_0^2}{\bar{a}^2}\right), \tag{6}$$

$$\bar{a}''+\bar{a}+\beta^2\bar{a}\left(1-\frac{\bar{a}_0^4}{\bar{a}^4}\right)=\bar{a}\left(\frac{1-\bar{a}_0^2}{1-\bar{a}^2}\right)^{1/2}, \tag{7}$$

where the symbol $'$ stands for derivative with respect to $\bar{\xi}=\Omega\xi$, and $\beta=\bar{k}/\Omega=(\bar{\omega}^2/\bar{k}^2-1)^{-1/2}$. The first integral of Eq. (7) reads

$$(\bar{a}')^2-(\sqrt{1-\bar{a}_0^2}-\sqrt{1-\bar{a}^2})^2+\beta^2\frac{(\bar{a}^2-\bar{a}_0^2)^2}{\bar{a}^2}=K. \tag{8}$$

The topology of the space $(\bar{a},\bar{a}')$ of Eq. (8) depends on the value of the parameter β. For $\beta=0$, the origin of the phase plane, $(\bar{a}=0,\bar{a}'=0)$, corresponds to an O point, the points $\bar{a}=\pm\bar{a}_0$, $\bar{a}'=0$ to X points, and the separatrix connects the two X points together [see Fig. 1(a)]. For $\beta\neq0$, Eq. (8) diverges on the axis $\bar{a}=0$, and the fixed point are $\bar{a}=\pm\bar{a}_0$, and $\bar{a}=\pm\bar{a}_1$, solutions of the equation

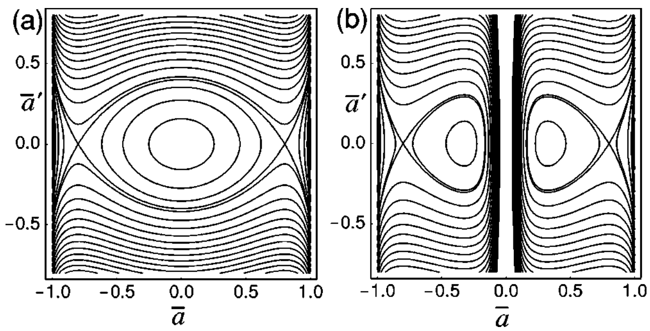

FIG. 1. Contours of Eq. (8) in the phase space $(\bar{a},\bar{a}')$, for $\bar{a}_0=0.8$ and $\beta=0$ (a), and $\beta=0.01$ (b).

$$\beta^2\left(1-\frac{\bar{a}_0^4}{\bar{a}^4}\right)+1=\left(\frac{1-\bar{a}_0^2}{1-\bar{a}^2}\right)^{1/2}. \tag{9}$$

They correspond to X points and O points, respectively, when condition $\mathcal{A}_0^2>0$ is fulfilled (and $|\bar{a}_0|<|\bar{a}_1|$), and vice-versa, where $\mathcal{A}_0^2=\bar{a}_0^2-4\beta^2(1-\bar{a}_0^2)$. In this case, there are two separatrix, one in each semiplane, which make a loop around the O points [see Fig. 1(b)]. From the analysis of the phase space, it is found that the localized solutions, which correspond to the trajectory on the separatrix, are "black" solitons for $\beta=0$, (i.e., solutions of "kink" type), and "gray" solitons for $\beta\neq0$. The solution is obtained integrating Eq. (8) with $K=0$, and can be written implicitly as

$$\pm\bar{\xi}=\frac{1}{\sqrt{1+\beta^2}}\sin^{-1}\mathcal{A}\left(\frac{1+\beta^2}{1+\beta^2\bar{a}_0^2}\right)^{1/2}+2\frac{\sqrt{1-\bar{a}_0^2}}{\mathcal{A}_0}\tanh^{-1}\frac{\mathcal{A}_0}{\mathcal{A}}\times\frac{\sqrt{1+\beta^2\bar{a}_0^2}-(1+\beta^2)\sqrt{1-\bar{a}^2}-\beta^2\sqrt{1-\bar{a}_0^2}}{\sqrt{1+\beta^2\bar{a}_0^2}-(1+2\beta^2)\sqrt{1-\bar{a}_0^2}}, \tag{10}$$

where the function $\mathcal{A}^2=\bar{a}^2-\beta^2[(1-\bar{a}_0^2)^{1/2}+(1-\bar{a}^2)^{1/2}]^2$ has been introduced. The phase θ can be obtained in terms of $\bar{a}$ integrating Eq. (6). Note that for $\beta=0$ the phase θ is constant so that the phase term for the wave vector potential is simply $\exp(-i\bar{\omega}\tau+i\theta_0)$ with $\bar{\omega}=\Omega$, i.e., in the moving frame the wave oscillates in time with the Doppler shifted frequency equal to the nonlinear plasma frequency Ω. For $\beta\neq0$, the phase θ reads

$$\theta-\theta_0=+\sin^{-1}\frac{\mathcal{A}}{\bar{a}}-\frac{\beta}{\sqrt{1+\beta^2}}\sin^{-1}\mathcal{A}\left(\frac{1+\beta^2}{1+\beta^2\bar{a}_0^2}\right)^{1/2}. \tag{11}$$

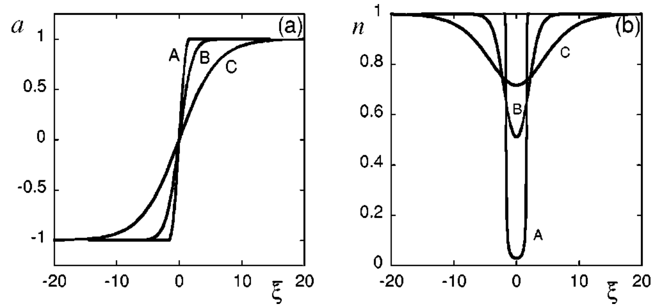

FIG. 2. Behavior of the vector potential amplitude a (a) and of the density n (b) as a function of $\overline{\xi}$, for a "black" soliton with $\beta=0$. The chosen parameters are $a_0=1$, and $V=0.02,0.5,0.9$ (curves A–C).

In this case, the phase θ changes from $\theta_0\pm S/2$ to $\theta_0\mp S/2$ as ξ goes from $-\infty$ to $+\infty$, S being the total phase shift across the soliton.

In the low amplitude case $(\overline{a}_0,\overline{a}\ll 1)$, the solution of Eq. (8) reads $\overline{a}^2=\overline{a}_0^2\tanh^2(\kappa\overline{\xi})+4\beta^2\,\mathrm{sech}^2(\kappa\overline{\xi})$, where $\kappa=(\overline{a}_0^2/4-\beta^2)^{1/2}$, with $\beta^2<\overline{a}_0^2/4$. From the above expression, it is easily seen that the solution is the standard kink soliton ("black" soliton) for $\beta=0$, and the "gray soliton" for $\beta\neq 0$, with minimum value $\overline{a}(\xi=0)=2\beta$. In the same limit, the phase θ is given by $\overline{k}\tan(\theta-\theta_0)=\kappa\tanh(\kappa\overline{\xi})$ for $\beta\neq 0$.

From the expression for the fluid quantities, it is found that the momentum $p_{x\pm}$ is always negative, and that the density is always less than 1 (density hole). In the center of the soliton at $\xi=0$, the modulus of the vector field amplitude is minimum, together with the density and the momentum. In Figs. 2 and 3, the e.m. wave amplitude a, and the density are plotted versus $\overline{\xi}$, for a fixed value of a_0, and different values of β and V, for the case of "black" and "gray" solitons, respectively. For fixed a_0 and V the minimum values are found in the case of the "black" soliton, i.e., at $\beta=0$. We see that for decreasing velocities, the width of the soliton decreases (in the low amplitude approximation as κ^{-1}), while the density profile inside the soliton becomes deep, and its value very low but finite. The particles velocity increases in modulus at the same time.

We note that the full problem of the soliton stability (both dark and bright soliton stability in electron-ion and electron-positron plasmas) has not yet been investigated analytically, starting from the full set of the equation including the study of the backward and forward stimulated Raman scattering. However, the stability problem can be solved in the framework of the envelope approximation as described in [12]. To this goal, we note that if, instead of the previous definition for $A_\perp$, we assume $A_y+iA_z=A(\xi,\tau)\exp(-i\omega t+ikx)$ with A complex and approximate the wave operator as $\partial^2_{xx}A_\perp-\partial^2_{tt}A_\perp\approx\partial^2_{\xi\xi}A+2i\overline{\omega}\partial_\tau A+(\overline{\omega}^2-\overline{k}^2)A$, the following nonlinear Schrödinger (NLS) equation for A is obtained

$$\partial^2_{\xi\xi}A+2i\overline{\omega}\partial_\tau A+\Omega^2A-AF(|A|^2)=0. \qquad (12)$$

In Ref. [12], dark solitons are defined as a solution of Eq. (12) of type $A=a(\eta)\exp[i\theta(\eta)]$, where $\eta=\xi-u\tau$, and u is a velocity. After normalization, it is found that a, and θ satisfy Eqs. (6),(7) in the variable η, where now $\beta=u\overline{\omega}/\Omega$. The stability criterion for dark solitons [12] reads $dP/d\beta>0$, where $P=-\beta\int_{-\infty}^{+\infty}d\eta(a^2-a_0^2)^2/a^2$ is the renormalized momentum associated to the NLS equation (12). By direct analytical integration, it can be shown that the dark solitons described by Eq. (10) are stable for any value of the amplitude and velocity. The explicit expression for $dP/d\beta$ is rather cumbersome, and it will not be presented here.

The dark solitons discussed above are regions of high gradient $(\approx\kappa a_0)$ of the electromagnetic field that can propagate with a speed arbitrarily close to the speed of light in vacuum. The radiation pressure of the soliton can accelerate charged particles depending on the phase of the wave-particle interaction. The particles can be injected into the soliton due to numerous reasons, e.g., thermal effects (not discussed in the present paper) as well as nonlinear wave breaking due to propagation of the soliton in another plasma. Furthermore, a sufficiently low density electron beam loaded in a proper place can supply the electrons to be further accelerated, without, at the same time, perturbing the soliton parameters. For example, in the case of the laser wake field accelerator, the wake-wave is stable and the injection of the charged particles is considered to be provided by some of the above mentioned mechanisms (see Refs. [15]).

To compute the energy variation, we write the Hamiltonian of a test particle in the field of the soliton, neglecting the back reaction of fast particles on the soliton itself. Let us denote by $\boldsymbol{P}$ the particle canonical momentum, conjugate to the variable $\boldsymbol{x}$. The Hamiltonian reads

$$H(\boldsymbol{x},\boldsymbol{P},t)=(1+|\boldsymbol{P}\mp\boldsymbol{A}|^2)^{1/2}.$$

Introducing $P_y+iP_z=P_\perp\exp(i\alpha)$, which is a constant of motion, and by means of a canonical transformation, with $\zeta=x-Vt$, $P_\parallel=P_x$, we get

$$H(\zeta,P_\parallel,t)=\sqrt{1+P_\parallel^2+|P_\perp\mp a(\zeta)\exp[-i\psi(\zeta,t)]|^2}-VP_\parallel, \qquad (13)$$

with $\psi=(\omega-kV)t-k\zeta-\theta(\zeta)+\alpha$. This Hamiltonian represents the test particle energy in the reference frame in which the soliton is at rest. It has $(1+1/2)$ degrees of freedom, and can exhibit a complex behavior. In the following, we analyze in detail the case for $P_\perp=0$ and $k=\omega V$ (i.e., the "black" soliton), leaving the analysis of the full problem to a future

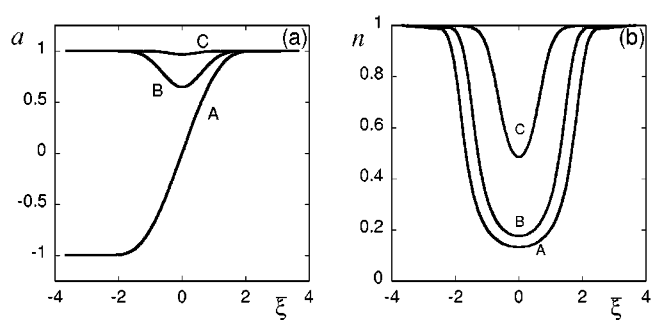

FIG. 3. Same as in Fig. 2 for a "gray" soliton. The chosen parameters are $a_0=1$, $V=0.1$, and $\beta=0,0.5,5$ (curves A–C).

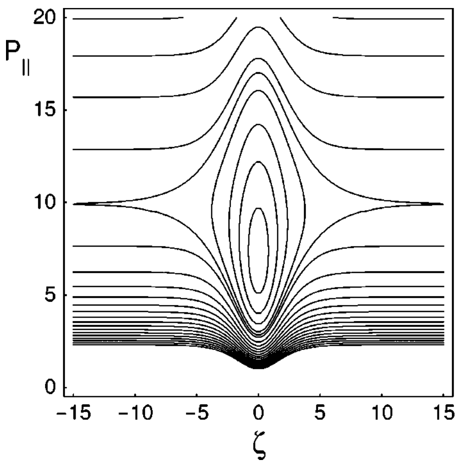

FIG. 4. Phase plot of the Hamiltonian system (14) in the $(\zeta, P_\parallel)$ plane, for $a_0=1$, $\beta=0$, and $V=0.99$.

investigation. In this specific case, the particle motion is integrable since the Hamiltonian becomes time independent

$$H(\zeta, P_\parallel) = \sqrt{1+P_\parallel^2+a^2(\zeta)} - VP_\parallel . \tag{14}$$

Note that the energy variation is simply proportional to the momentum variation, $\delta\gamma = V\delta P_\parallel$. The phase plot that corresponds to constant values of the Hamiltonian (14) is shown in Fig. 4. In the $(\zeta, P_\parallel)$ plane, there is an O point at $\zeta=0$, $P_\parallel = V/(1-V^2)^{1/2}$, at $H=H_O=(1-V^2)^{1/2}$, and two X points at $\zeta=\pm\infty$, $P_\parallel = V[(1+a_0^2)/(1-V^2)]^{1/2}$, and $H=H_X=[(1-V^2)(1+a_0^2)]^{1/2}$. For $H_O<H<H_X$, the trajectories correspond to particles that are trapped in the field of the dark soliton. We see that the dark soliton contrary to the bright soliton can advect trapped particles. While the particles bounce inside the soliton their momentum (and energy) changes. The highest momentum is gained by the particles with the trajectory close to the separatrix. It varies between minimum and maximum values given by $p_{\rm min,max} = [V(1+a_0^2)^{1/2} \mp a_0]/(1-V^2)^{1/2}$. In the limit $a_0 \gg 1$, we obtain $p_{\rm max} = a_0[(1+V)/(1-V)]^{1/2}$.

In conclusion, the problem of the existence of coherent nonlinear structures in the pure electron-positron pair plasma described by the full set of relativistic hydrodynamics equations and Maxwell equations has been solved. We have demonstrated that the natural nonlinear localized mode in the electron-positron plasma is the dark soliton. The electron-positron plasma in this case exhibits properties similar to those in the Bose-Einstein condensate with the positive scattering length [16]. Inside the dark soliton, the plasma is electrically neutral, this condition being exact, contrary to the case of the solitons in the electron-ion plasma in the long-wavelength quasineutral approximation. The dark soliton corresponds to the minimum of the electromagnetic energy density and to the minimum of the plasma density, and propagates without change of its form with velocity arbitrarily small or arbitrarily close to the speed of light in vacuum. The dark soliton has a continuous spectrum contrary to the bright solitons in the electron-ion plasma [17]. We note here that in the electron-ion plasma, dark solitons are found in a range of very low propagation velocity, $V \lesssim \sqrt{m_e/m_i}$ [18]. We have also shown that the dark soliton can advect the particles trapped inside effective wells formed by the radiation pressure. The trapped particles can gain an energy substantially higher than the kinetic energy of the particles of the background plasma inside the soliton. The bunches of fast particles correlated with the variations of the electromagnetic field provide an observational signature of the dark solitons.

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